Validation and Verification of Multi-Agent Systems

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PREPRESS VERSION

5.1 INTRODUCTION

Use of multi-agent system models is moving from being an esoteric technique towards becoming mainstream research in a variety of disciplines. With general acceptance comes the responsibility of ensuring that multi-agent systems are subject to procedures commonly accepted for other kinds of modeling techniques. In particular, there is room for better validation and verification. These terms concern, respectively, the truthfulness of a model with respect to its problem domain and the correctness of its construction. In other words, verification means “building the system right” and validation means “building the right system.”

This chapter discusses validation and verification for multi-agent system models of socio-ecosystem dynamics. It begins by examining general criteria by which a multi-agent system model may be measured. The chapter continues with several sections on the challenges raised by the verification and validation of multi-agent system models. The chapter next explores issues of complexity, scale, and emergence, all of which are of particular importance to the validation and verification of multi-agent systems. The chapter concludes with a discussion of general themes and specific research directions.

GENERAL CRITERIA FOR VALIDATION AND VERIFICATION

Most of the multi-agent system models in this volume share roughly similar development paths and therefore can be considered via a common terminology. Most modeling efforts have underlying theory that is distilled into a *conceptual model*. The modeler then uses *calibration data* to instantiate the conceptual model in a *software model*, in particular, a *multi-agent system* *model* that represents a *target system*. The multi-agent system model is then subject to *verification*, which involves testing the software model to ensure the proper functioning of its underlying programming. The multi-agent system model is then run in order to create *model outcomes*. *Validation* comes in two varieties, *structural validation*, or how well the software model represents the conceptual model, and *outcome validation*, how successfully model outcomes characterize the target system.

Just as verification and validation are two facets of a larger modeling enterprise, they are two ways of answering a general question asked of all models: how well does a model characterize the target system? There are a number of general criteria by which model success may be evaluated (Kwasnicki 1999) and each bears on validation and verification of multi-agent systems:

Correctness: model structure and outcomes must be similar to those of the target system.

Consistency: the model must be internally consistent and match the conceptual framework in order to describe the target system.

Universality: the model should be applicable to circumstances beyond those described by the calibration data.

Simplicity: when choosing between two models, all other things being equal, the less complicated model is preferable.

Novelty: a model should create new knowledge or outcomes.

Correctness bears directly on validation. How well does the model achieve its goals? These goals often focus on having the model reproduce behavior or phenomena found in the target system. In general, validation is achieved by comparing model structure and outcomes to pertinent characteristics of the target system. In order to accomplish this task, one has recourse to a number of techniques, explored below.

Consistency and *simplicity* are essential to understanding how well the model articulates the conceptual framework. In particular, how well do the various model components fit together to represent the ideas that underlie them? In this vein, a simpler model may be a better one. If the modeler can reduce the number of modules or lines of code without changing model function, then the model is arguably better since there are fewer opportunities for model artifacts. Similarly, the conceptual framework is more easily understood when it has fewer objects and linkages. There is also typically a declining marginal rate of return on making a model more complex in an attempt to better mirror reality due to error propagation and making the model less generalizable (Alonso 1968).

Generalization, or *universality*, is a function of model construction. When a model is closely tailored to particular conditions, its ability to speak to other conditions is reduced. By definition, a model is not an exact representation of the target system. It follows that if a model is only applicable to the narrow confines of the data on which it is calibrated, the model may fail to represent changes in the target system. Since most socio-ecosystems are dynamic, models of these systems must retain some universality.

Novelty poses a dilemma to modeling. In a general sense, if one has calibration data for a target system with which to create a model, then that phenomenon is known to the modeler and therefore not novel. In a more specific sense, however, generalizable models should be able to produce novel outcomes if they are fed data apart from that used for calibration. This is particularly so for a model that is well verified and validated, since its conceptual framework likely addresses situations beyond that represented by calibration data.

5.3 DATA

Verification and validation involves ensuring a model functions well and comparing its structure and outcomes to validation data. The nature of this data varies from observations of the target system to the products of another model or theory assumed to adequately characterize the target system. For socio-ecosystems with significant human intervention, particularly valuable 'real' data are found through surveys, interviews, censuses, and broader scale remote sensing or geographic information sources. Validation data may also be taken from theory or the outcomes of other models. While this data is arguably not as realistic as other observations, many phenomena that are of interest to multi-agent system researchers are not readily measured. Modelers must look to all available sources of validation data.

For models meant to project future trends, validation data should not be used in model construction. There is a fundamental, data-driven relationship between validation and calibration. Studies of land-use and land-cover change, for example, are considered fortunate when they have data for one time, let alone data for two or three periods (Mertens and Lambin 2000). There is an understandable desire to use all data for model construction and calibration that is, unfortunately, at cross-purposes to the necessity of holding back data for validation. In the end, some data should be held back from construction for use in validation.

If a model is less focused on projection and more towards shedding light on a theoretical question or explaining a phenomenon, then outcome validation may not be as important as verification and structural validation. There is therefore less need to reserve validation data. The structure of the model may be the ‘outcome’ of interest since the model does not produce outcomes in the form of projections. The most common example of this distinction is the use of all available data for calibration of regression-based models (e.g., Mertens and Lambin 2000). In this case, the model is less subject to outcome-validation and more so to verification and structural-validation with measures such as goodness-of-fit or expert opinion.

5.4 VERIFICATION

Multi-agent system models are flexible in their specification and design, which can lead to models that are difficult to manage. Verification reduces the problematic nature of flexibility by vetting model structure. Verification lies largely in forcing the model’s underlying mathematical and computational components to fail by varying model configurations according to all foreseeable model inputs.

This ‘breaking’ of the model for verification purposes is similar to sensitivity testing, in which parameters are varied across repeated model runs in order to observe changes in simulation performance. Of particular interest is determining the spatial or temporal bounds of model applicability (Klepper 1997). Sensitivity tests map incremental parameter changes against model outcomes to determine the influence of model structure on outcomes.

Two chapters in this volume, Lynam (2003) and Deffuant et al. (2003), offer prime examples of sensitivity analysis. As the latter notes, there can be too many model parameters to fully explore model sensitivity by incrementing variables. By using an appropriate sample design, however, key thresholds such as one standard deviation can be employed to give a good sense of where the model performs well. As the former work notes, it is also possible to break model results into clusters by use of a decision tree to classify results. The decision tree indicates the sensitivity of the model to parameters and allows the users to cluster parameter values.

5.5 IMPORTANCE OF DETAILS

Regardless of the exact means of verification, the overall process leads to careful examination of model objects and the linkages among them. Verification of multi-agent system models necessitates exploration of subtle subjects such as the effects of different randomization techniques. Ruxton and Saravia (1998), for instance, demonstrate how spatiotemporal ordering of events can create model artifacts. These sorts of errors are more likely in multi-agent systems given the potential for nonlinearities. The importance of space is also illustrated by several chapters in this volume, (e.g., Bousquet et al. 2003; Deadman and Lim 2003; Hoffman et al. 2003; Janssen et al. 2003; Lynam 2003), where inclusion of spatial heterogeneity in a multi-agent system results in fundamentally different model behavior.

Even when attention is paid to order, it is possible to have slight, cumulative errors introduced into a model from seemingly well-established tools such as random number generators (Stroustrup 1998). Error propagation and uncertainty are topics often left unconsidered in modeling (Robinson 1994). Error propagation can be estimated by examining the kinds of operations performed on data (Alonso 1968). Other approaches include treatment of error and uncertainty in geographic information systems within a Monte Carlo framework (Heuvelink and Burrough 1993; Eastman 1999) or studies of thematic error propagation (Veregin 1995). Particularly promising is use of Bayesian analysis to quantify a priori beliefs on the part of the researcher (Berger 1986). Bayesian analysis proves useful in addressing the impact of error and uncertainty in both model parameters and model structure (Berk 1994).

5.6 NORMALITY AND COMPLEXITY

Verification and validation of multi-agent system models is characterized by two general and sometimes conflicting themes. First, in order to use the bulk of statistical and mathematical methods, models should act in ways that support them. Model outcomes should change roughly in proportion to changes in inputs. Similarly, while a model cannot be expected to perform well with all sets of inputs, outcomes should gracefully degrade with incremental change in parameters away from calibration data. Proportionality and graceful degradation are key to statistical methods of verification. Measurement error, sampling regimes, observer bias, and data availability – all of these factors also feed into verification. A model should accommodate the imprecise nature of the world and the data with which it is characterized. The statistical characteristics of model operation are manifest in many assumptions of verification techniques.

The second theme is contrary to the first since multi-agent systems should be able act as complex systems and experience large shifts in outcome in response to small changes in inputs. These shifts come from sources such as sensitivity to initial conditions or self-organized criticality (Manson 2001). Interdependencies may create large, sudden shifts in model behavior due to small changes in parameter values. For many modelers, the potential for complex behavior is an important reason to use multi-agent systems.

Consider how complexity affects sensitivity analysis. In addition to the methods examined above, one can use an active nonlinear test (ANT) to discover model failures that result from nonlinearities (Miller 1998). Since multi-agent system models are often meant to simulate complex, non-linear systems, they may not be amenable to traditional testing that modifies one parameter at a time. An ANT is an automated test that seeks out sets of strongly interacting parameters in a search for nonlinear relationships across variables that are not found by traditional sensitivity testing.

There is a key caveat, however, in considering the effects of complexity on verification and validation. Not all models will behave in a way that suggests that complexity or strong nonlinearities are present. The critical point is that sudden changes should be possible, but they must also be explained by the conceptual framework. It is not enough to look at a new, potentially surprising outcome and treat it as an outlier. Outcomes must be subject to verification and validation so they can be traced back to conceptual precursors and not poor model design.

5.7 SPACE AND TIME

Statistics is home to an array of techniques geared towards description and hypothesis testing, most of which is applicable to validation of multi-agent systems. In addition to standard statistical tests, the complexity of both ecological and multi-agent systems suggests the use of tests that measure spatiotemporal outcomes (Turner et al. 1989). This need is evident in use of spatial statistical approaches to compare modeled outcomes to validation data in a variety of multi-agent system applications (e.g., Alberti and Waddell 2000; Batty 2000; Manson 2000; Hoffman et al. 2003).

Hoffman et al. (2003) and Janssen et al. (2003) note the importance of having spatial components in socio-ecosystem models, which in turn necessitates use of spatial validation tools. In the former chapter, land-use and land-cover are key components of, and outcomes from, agent processes. It notes the importance of matching both spatial and temporal outcomes to data and experience garnered from the target system. This also holds true for examining aggregate changes over time, such as rates of deforestation and afforestation; when exploring the causality inherent in spatial inputs, such as the relationship between slope and agriculture; and finally, more in-depth measure such as pattern and ecological landscape function.

A number of issues are raised by even seemingly simple validation metrics. By way of example, consider the simplest class of spatial validation techniques, which act on model outcomes in the form of mutually exclusive categorical map layers (e.g., Gilruth et al. 1995). While useful, these error matrices are flawed in that the proportion of correctly classified cells is difficult to interpret since they may be classified correctly due to chance. The Kappa statistic accounts for this problem (Hudson and Ramm 1987) but as noted by Pontius (2000), Kappa fails to recognize when model outcomes have accurately determined the relative quantities of each cell state. He therefore introduces a new Kappa measure that differentiates between location and quantity.

Pattern indices offer another example of how even seemingly simple validation tools can be problematic. These metrics include dominance, patch size variance, fractal dimension, nearest neighbor probabilities, contagion, and adjacency (Turner et al. 1989; Giles and Trani 1999). These pattern indices roughly translate to the statement "if the state of a cell is known, by how much is uncertainty about neighboring cells reduced?" Ecologists use fractal dimension and its various offshoots to understand landscape ecologies (Krummel et al. 1987). Fractal measures are also used to validate model outcomes but these statistics must be applied with care due to the effects of resolution, scale heterogeneity, and the uncertain links between fractal metrics and ecological function (Li 2000). These issues are emblematic of the problems of projecting biotic diversity in multi-agent system models, as noted by Abel (2003) in this volume.

5.8 STRUCTURE AND REALITY

On the boundary between calibration, validation, and verification is the issue of how well a software model maps onto the cognitive model that underlies it. A completely inductive 'black box' model may produce seemingly valid outcomes but it is not readily verifiable or structurally valid if causal mechanisms are not adequately represented. This becomes a problem when a structural change in the target system renders the model less able to produce valid outcomes.

Bousquet et al. (2003) illustrate how a multi-agent system, CORMAS, is calibrated and verified through involvement of stakeholders. Over repeated interactions between stakeholders and modelers, the objects and rules that make up the software model are more closely mapped onto the target system. Repeated examination of the fit between the model and target system makes model failures more apparent; verification and structural validation are therefore more easily achieved.

Deffuant et al. (2003) present a multi-agent system model of the diffusion of agri-environmental measures oriented towards understanding the role of uncertainty, the importance of information, and the decision-making process. In their model, the authors use real data for both structural and outcome validation. Another chapter in this volume, Jager et al. (2003) details how experimental data can be used to both calibrate and validate a multi-agent system model. Here the focus is on the use of experimental laboratory data on common-pool dilemmas for validation of multi-agent models.

When used with both theory and real data, multi-agent system models serve as a locus of theory, experiment, and modeling. All three can posit different behavior. Such is the usefulness of multi-agent systems for understanding the dynamics of socio-ecosystems, whether real, artificial, or theoretical. The most cogent result of validation may be illustrating the need for more validation data since this need can define how further experiments should be conducted. As a result, in addition to looking to theory for hypothesis construction, we can look to multi-agent systems. This holds especially true in terms of narrowing down the host of all possible options and parameters to a manageable number.

5.9 OUTCOMES AND THEORY

Multi-agent system models are also concerned with abstract concepts, which affects validation and verification. Agent-based computational economics, for instance, is concerned with the "constructive grounding of economic theories in the thinking and interactions of autonomous agents" and "the formulation and testing of conceptually integrated socioeconomic theories compatible with theory and data from many different relevant fields of social science" (Tesfatsion 2001: 283). In this case, modeling is driven by theory in order to affirm theory. Also noted, however, is the importance of validating models with "analytical studies, econometric studies, field studies, and human-subject laboratory studies" (ibid).

Balmann et al. (2003) compare model results, in this case farm characteristics, to general characteristics of the target system. The model is calibrated with survey data and then model structure is compared to data from a real-world system of agriculture. In terms of outcome-validation, it does not involve comparing spatial patterns, as above, but it is concerned with comparing model statistics with idealized, yet realistic, characteristics. Real-world processes such as specialization, diversification, and stratification are recreated in the model outcomes. Similarly, model scenarios are created to match policy scenarios advocated in reality. These scenarios can then be examined from a number of stylized, theoretical perspectives to see if they are qualitatively reasonable.

In many respects, the data-theory dialectic lies at the heart of validation. While multi-agent system modelers are often interested in adding another dimension to our understanding of a target system, they are also interested in challenging the status quo. Two chapters in this volume use different theoretical models in a form of validation (Jager and Janssen 2003; Janssen 2003). Both demonstrate the importance of theory to validation when examining the evolution of institutions. They note there is a long-standing body of theory on common pool resources, exemplified by Hardin's "tragedy of the commons" (1968). A modeler could draw on this body of work in order to validate a multi-agent system model of common pool interaction. As both chapters note, however, experimental evidence suggests that there are instances in which the theory is incorrect (also see Feeny et al. 1990). In this case, its use as a source of validation data would be potentially incorrect. Both chapters also note how multi-agent system models highlight how individual agent behavior can be at odds with theory, particularly when there is explicit interaction between agents.

5.10 SCALE AND MULTI-AGENT SYSTEMS

The bulk of the statistical methods for validation and verification described above suffer from three scale-related problems that affect their use for multi-agent systems. First, if the resolution, or granularity, of a model outcome is divided into exclusive categories, then other outcomes may be separated from processes not directly mapping onto this resolution. Using watersheds to define spatial resolution, for instance, makes it difficult to examine phenomena not tied to hydrology. Second, if a given resolution is achieved through aggregating finer-grained phenomena or objects, then there is the potential to treat units at this level as just simplifications of the those found at lower levels and not as entities that can have their own behavior (Kimble 1951). This problem is especially pertinent to validating collective behavior in agent-based models.

A final scalar challenge lies in the fact that the validation statistics noted above are impacted by changing resolution (Lam and Quattrochi 1992) and extent (Saura and Millan 2001). Ecological fallacy occurs when the characteristics of an individual are incorrectly inferred from those of the population from which it is drawn (Robinson 1950). The related modifiable areal unit problem (MAUP) is the sum of an aggregation effect, when larger spatial groupings of data create better correlations, and a zoning effect, whereby an area can be sub-divided into an almost infinite array of configurations that all share a common statistic, such as area, yet differ in others (Openshaw 1977).

Ecological fallacy and MAUP can be statistically causal when covariation between variables is affected by the scale at which they are compared. In essence, scale can become an ill-chosen independent variable (Bian 1997). Research in land-use and land-cover change, for instance, has demonstrated that changing resolution can affect the apparent magnitude and direction of relationships among causal factors of land conversion (Kummer and Sham 1994). Various remedies for the statistical effects of scale have been proposed, including inductive search techniques (Openshaw et al. 1987), fractals (Lam and Quattrochi 1992), and geostatistics (Chou 1993).

5.11 EMERGENCE

In addition to potential problems faced by statistical approaches because of scale, challenges are offered by the concept of emergent hierarchies. The term *emergent* refers to the fact that a system can have qualities that are not analytically tractable from the attributes of its internal components (Baas and Emmeche 1997). Emergence is a function of synergism, whereby system-wide characteristics do not result from superposition but instead from interactions among components (Lansing and Kremer 1993). An economy has emergent qualities such as volatility and investor ‘herd behavior’ that are commonly attributed to irrationality or imperfect markets but in fact are intrinsic to rational, local, interactions (Andreoni and Miller 1995).

The notion of emergence is a central tenet of multi-agent system modeling. Emergence challenges accepted scale levels. Economics, for instance, studies stability and repeated patterns, while perspectives that employ emergence examine the evolution and change of levels (Lane 1993). Similarly, institutions of importance to global change are seen as coming into being, or emerging, from the rational actions of individuals (Ostrom et al. 1994), a theme echoed by Janssen (2003).

Emergent hierarchies illustrate the importance of blending quantitative and qualitative methods of validation for multi-agent systems. Consider the challenge of abstraction. Many attributes and outcomes of human interaction, such as trust or learning, are imputed or abstract. As noted earlier, validating such abstract outcomes may be difficult since they are not easily measured. Validation is even more difficult if outcomes are simultaneously abstract and emergent since they are hard to define in addition to being hard to measure.

At this point, the modeler may be left with only large-scale or qualitative measures of model performance. Balmann et al. (2003), for instance, draw on similarities between modeled outcomes and real, aggregated measures such as structural change or economic phenomena such as economies of scale and income disparities. Bousquet et al. (2003) address the challenges of abstract and emergent outcomes with expert and stakeholder interviews that provide a sense of how these outcomes are related to model structure. The process described by the authors could conceivably be applied to very rare events projected by a model. Since it may be impossible to draw upon reality to ascertain the validity of an outcome, one is left to interview experts to determine its validity.

5.12 CONCLUSION

By way of conclusion, Hoffman et al. (2003) refer to a validation scale that has four levels that range from a model being able to generate micro patterns that quantitatively match empirical patterns to a model possessing agents whose behavior qualitatively matches agents in the target system. The schema presents verification and validation as ranging from standard statistical techniques, through those that measure more nuanced characteristics of outcomes, to more qualitative techniques that require iterative, structured exploration by stakeholders. The key point of this schema and the discussion above is that no one technique is the best, or even adequate, when used by itself. Good verification and validation requires use of multiple, complementary methods.

The various challenges faced in model validation and verification highlight the need for more sophisticated methods. In creating these methods, multi-agent system modelers can consider two broad questions. First, what does the modeler learn when different model configurations enjoy varying levels of success across different forms of verification and validation? Is a significant difference in one metric between configurations due to a difference in approach or the metric itself? Second, does a model behave as expected when key components or their interdependencies are varied? This question goes to the heart of the modeling process in that a model is only as strong as its underlying theory. It is the role of both verification and validation to determine which components are important and why.

In the meantime, verification and validation of multi-agent systems will become easier with better communication of model design. The appropriateness of verification and validation procedures is difficult to establish when publications do not describe model design in sufficient detail to permit the reader full understanding. Given the necessarily restricted scope of most research papers, however, it can be difficult to fully elucidate model design, although a number of chapters in this volume do a good job of conveying model structure with mathematics and graphics. A growing tradition of publishing software code along with manuscripts exists within the agent-based modeling community.

Similarly, adoption of common languages would allow streamlined communication of model parameters and development of common validation and verification tools. The growth of SWARM, REPAST, CORMAS, and other frameworks is heartening, but there is much room for improvement, particularly in terms of linking these packages to other kinds of software, such as geographic information systems. As evidenced by several chapters in this volume, there is still a need to create custom modeling tools that in turn affects the extent to which multi-agent system are subject to validation and verification.

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